

Competition and the Strategic Disclosure of Innovation

Theory and Evidence from Patent Applications

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Roadmap

Motivation

Model

Equilibrium Results

Comparative Statics and Predictions

Empirical Evidence

Conclusion

Motivation: Secrecy vs. Disclosure

- Secrecy is one of the most important strategies to protect inventions
 - More popular than patents (Levin et al., 1987; Cohen et al., 2000)
 - Mandatory disclosure is costly (Kim and Valentine, 2021)
- Yet firms *voluntarily* announce their innovations
 - Press releases about pending patent applications
 - Mentions of filed patents in SEC filings
 - Requests for early publication at the USPTO
- When do firms find it profitable to disclose a pending patent application?

Motivating Examples

- ProPhotonix (2015): announced a pending patent application on a heat sink for optical modular array assemblies via press release
- PyroGenesis Canada (2015): announced a provisional patent for producing high purity silicon from silica via press release [Full quotes](#)
- Both are established firms (founded 1951 and 1991) → start-up funding is not the explanation
- Practitioner view (Gunderman and Hammond, 2007):
 - Competitors seeing “patent pending” wonder about the scope
 - “Fear of the unknown” provides a marketplace advantage

This Paper

- **Theory:** A duopoly signaling model where an innovative leader strategically announces a pending patent application
 - Announcement affects both *product-market competition* (short run) and *rival's R&D* (long run)
 - Reduced-form competition nests strategic substitutes and complements
- **Empirics:** Test predictions using three disclosure proxies
 - SEC 10-K filings mentioning pending patents
 - Press releases about pending patents
 - Early Publication Requests (EPRQs) at the USPTO

Preview of Results

- **Model predictions:**

- Classic result (Gal-Or, 1986): Disclosure optimal under strategic substitutes, non-disclosure under strategic complements
- **New:** When R&D correlation is low, announcement deters follower's R&D
 - disclosure even under some strategic complements
- **New:** When R&D correlation is high, announcement triggers follower's R&D
 - non-disclosure even under some strategic substitutes

- **Empirical findings:**

- More disclosure in industries with competition in strategic substitutes
- More disclosure when R&D correlation across firms is low
- Interactions as predicted by the model
- Consistent results across all three data sources

Contribution to:

1. **Information disclosure in oligopoly**

- Extends Gal-Or (1986), Darrough (1993), Hughes and Pae (2015)
- Framing: disclosure of pending patents, not production costs
- New channel: long-run effect on rival's R&D decision

2. **First empirical evidence** supporting the classic theoretical prediction that disclosure depends on the mode of competition

3. **Economics of innovation and IP management**

- Disclosure of the *existence* of a technology, not technical details
- Firms strategically exploit the statutory secrecy period of pending applications

4. **Discretionary disclosure and competition**

- Accounts for both the nature *and* the intensity of competition
- Extends Glaeser and Landsman (2021), Rathee et al. (2025)

Where This Paper Fits: My Research Agenda

My research: innovation economics — how do firms create, protect and disclose knowledge; how do legal institutions affect this; how to measure innovation.

1. Firms' strategic IP decisions: protection, disclosure, and enforcement

- *Exclusive Secrets* (with Calzolari & Naghavi): trade secret laws and supplier relationships — cond. accepted, *Journal of the European Economic Association*
- *This paper* (with Ganglmair): when firms voluntarily disclose pending patents
- *The Private Cost of Legal Uncertainty*: firms' valuation of the Unified Patent Court
- Future: supplier collusion as substitute for secrecy; patents without technology

2. Innovation measurement

- *Technology Gaps and Income Inequality* (with Mariuzzo & Zhang) — published, *Econ. Inquiry*
- *Trade Secrets in Patent Data*: disentangling disclosure from innovative effort

3. University–industry knowledge exchange

- Survey on firms' engagement with universities
- Survey experiment on knowledge sourcing (with Salter) — R&R, *Research Policy*

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Setup: Two Firms, Two Stages

- Two firms: technology **leader** L and **follower** F
- Leader *may* have access to a new (superior) technology
 - “Good” leader (G): has the new technology and a patent application
 - “Bad” leader (B): produces under the status-quo technology
 - **Leader’s type is private information**

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- Follower can invest in R&D to develop her own version
 - Same cost-saving or demand effect as leader’s technology
 - R&D success is uncertain, observed with delay

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- Follower can invest in R&D to develop her own version
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 - R&D success is uncertain, observed with delay
- Firms compete in the product market **twice**:
 - Stage 1: under asymmetric information (before patent is published)
 - Stage 2: under complete information (after patent is published)

Timeline of the Game

- Stage 1*
- $t = 0$: Leader has patent application (or not)
 - $t = 1$: **Good Leader's announcement decision** (announce or stay silent)
 - $t = 2$: **Follower's R&D investment decision**
 - $t = 3$: Stage-1 product-market competition

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 - $t = 2$: **Follower's R&D investment decision**
 - $t = 3$: Stage-1 product-market competition
- Stage 2*
- $t = 4$: Patent application published and examined
 - $t = 5$: Both firms observe follower's R&D outcome
 - $t = 6$: License negotiations
 - $t = 7$: Stage-2 product-market competition

Important feature: The leader's announcement can influence the follower *before* the patent office reveals the same information anyway.

The Announcement as a Signal

- Only a leader with a real patent application can credibly announce (assumption)
 - “Patent pending” marks, press releases, public statements
 - Reveals the *existence* of a technology (the “What?”), not the details (the “How?”)
- **If the leader announces:** follower learns for certain the leader has the new technology
- **If the leader stays silent:** follower becomes *more skeptical* that the leader has the technology (since a leader with good news *might* have announced)
- This is a classic signaling structure: the leader’s silence can itself be informative

Key Dimension 1: Nature of Competition (σ)

How does a rival's strength affect my profits?

- **Strategic substitutes** ($\sigma < 0$): a stronger rival makes me *less* aggressive
 - E.g., quantity competition (Cournot): if rival produces more, I produce less
- **Strategic complements** ($\sigma > 0$): a stronger rival makes me *more* aggressive
 - E.g., price competition (Bertrand): if rival cuts price, I cut price too

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Captured through L 's stage-1 profits as a function of F 's beliefs $\hat{\theta}_1$ about L 's type:

$$\tilde{\pi}_i(\hat{\theta}_1) = \begin{cases} \tilde{\pi}_B = (1 - \sigma \hat{\theta}_1) \pi_{BB} \\ \tilde{\pi}_G = (1 - \sigma \hat{\theta}_1 + \sigma) \pi_{GB} \end{cases}$$

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- Higher $\hat{\theta}_1$ (F more convinced L is strong) → profits decrease if $\sigma > 0$, increase if $\sigma < 0$

Key Dimension 2: R&D Correlation (τ)

How similar are the firms' R&D projects?

- Low τ : firms work on largely independent technologies
 - E.g., pharma firms targeting different biological pathways
- High τ : success of one firm predicts success of the other
 - E.g., semiconductor firms working on the same chip architecture

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Follower's R&D success probability given her beliefs $\hat{\theta}_1$:

Spillover model

$$\tilde{\theta}(\hat{\theta}_1) = (1 - \tau) \theta + \tau \hat{\theta}_1$$

- $\tau = 0$: success independent of leader's type → announcement uninformative for R&D
- $\tau = 1$: perfect correlation → follower succeeds iff leader is good type
- Intermediate τ : announcement shifts follower's expectations about her own R&D

Product-Market Competition (Reduced Form)

- We use a reduced-form approach that nests different competition models
- Key properties of equilibrium profits π_{ij} (firm i of type, facing firm j of type):

1. **Competitive advantage:** Having the new technology is always better

$$\pi_{GB} > \pi_{GG} > \pi_{BB} > 0 = \pi_{BG}$$

- Best case: I have the technology, my rival does not (π_{GB})
- Worst case: my rival has it, I do not ($\pi_{BG} = 0$)

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■ Best case: I have the technology, my rival does not (π_{GB})

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2. **Gains from trade:** Both firms benefit from knowledge transfer

■ Licensing is always mutually profitable: $2\pi_{GG} > \pi_{GB} + \pi_{BG}$

How Competition Affects the Value of Secrecy

- The follower's *beliefs* about the leader's type affect stage-1 competition
- The impact depends on the nature of competition:

Strategic complements ($\sigma > 0$)

Example: price competition

- If follower *knows* leader is strong $\rightarrow$
competes more aggressively
 - Leader's profits *decrease*
- $\Rightarrow$ Leader prefers to keep rival uncertain
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This is the **short-run effect** of disclosure — the classic insight from Gal-Or (1986).

How R&D Correlation Affects the Value of Deterrence

- Beyond the short-run product-market effect, the announcement also affects the **follower's R&D decision**. This is the **long-run effect** — our new channel.
- The follower has two reasons to invest in R&D:

IP and licensing model

 1. **Competitive advantage:** if the leader turns out to be weak, own R&D creates an edge
 2. **License-fee savings:** if the leader is strong, own R&D avoids costly licensing

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Low R&D correlation (τ small)

- Announcement reveals leader is strong, but says little about follower's own chances
- Follower is *deterred*: "the leader is ahead, and my R&D won't help much"

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High R&D correlation (τ large)

- Announcement reveals leader is strong, and follower infers *she too* is likely to succeed
- Follower is *encouraged*: "if they can do it, so can I"

⇒ Announcement triggers R&D

The Leader's Trade-Off

The leader balances two forces when deciding whether to announce:

1. **Short-run: value of secrecy** (stage-1 product-market competition)
 - Strategic substitutes ($\sigma < 0$): announcing helps (weakens F 's response)
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→ The equilibrium announcement strategy depends on the **interaction** of σ and τ .

→ When the two forces *align*, the prediction is clear. When they *conflict*, the outcome depends on which force dominates.

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Equilibrium: Solving the Game

- We solve for Perfect Bayesian Equilibria by backward induction:
 1. License fee determined by Nash bargaining: $\lambda = \frac{\pi_{GB}}{2}$
 2. Follower's R&D: invest if expected net benefits $R(\hat{\theta}_1) \geq 0$
 3. Leader's announcement: trade off short-run and long-run effects
- **Key condition:** does the announcement weaken or strengthen the follower's incentive to invest in R&D?
- This depends on whether the *competitive-advantage motive* for R&D dominates the *license-savings motive* (which itself depends on τ and the follower's R&D costs)
 - If competitive advantage dominates: learning the leader is strong *discourages* R&D
 - If license savings dominate: learning the leader is strong *encourages* R&D (to avoid fees)

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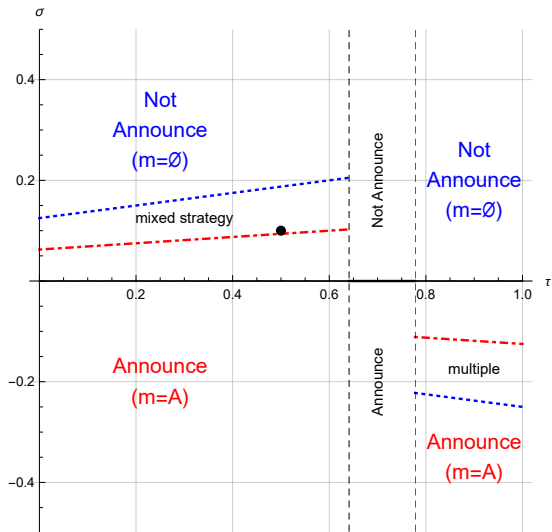
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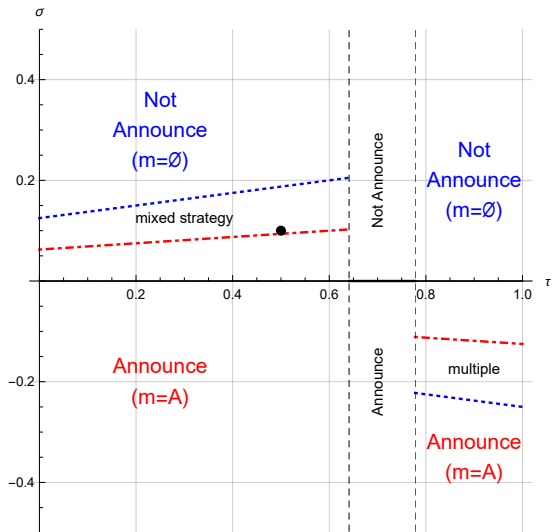
Equilibrium Outcomes and Predictions (1/4)

Full equilibrium



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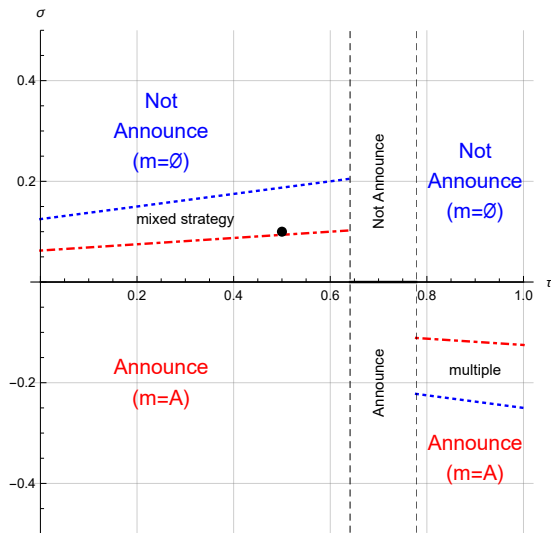


Reading the figure:

- **Left region: R&D deterrence** (low τ)
 - Follower's R&D incentives *decrease* after announcement

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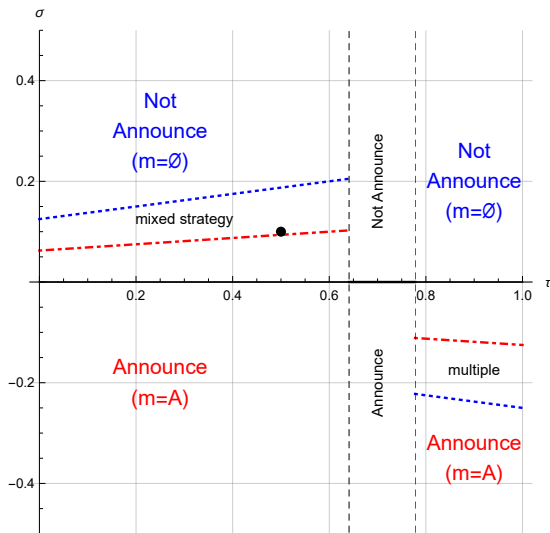


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- **Bottom: Announce** (substitutes)
 - Both forces favor disclosure: short-run competitive gain + R&D deterrence

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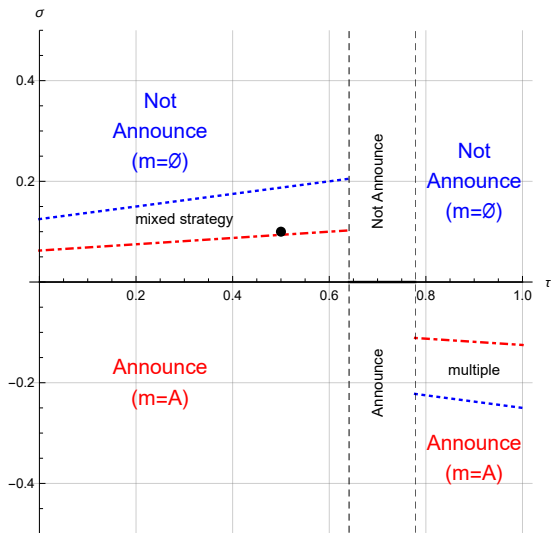
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Reading the figure:

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- **Bottom: Announce** (substitutes)
 - Both forces favor disclosure: short-run competitive gain + R&D deterrence
- **Top: Don't announce** (complements)
 - Short-run secrecy benefit outweighs R&D deterrence
 - **New insight:** Boundary shifts *upward* vs. Gal-Or

Equilibrium Outcomes and Predictions (2/4)

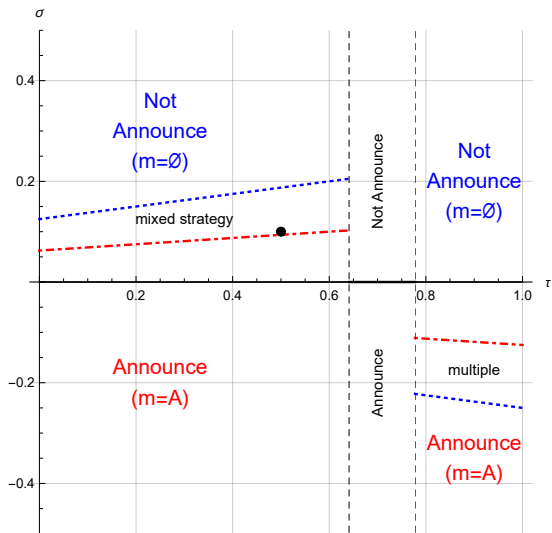


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Middle: Gal-Or benchmark (intermediate τ)

- R&D channel inactive; depends only on σ

Equilibrium Outcomes and Predictions (2/4)



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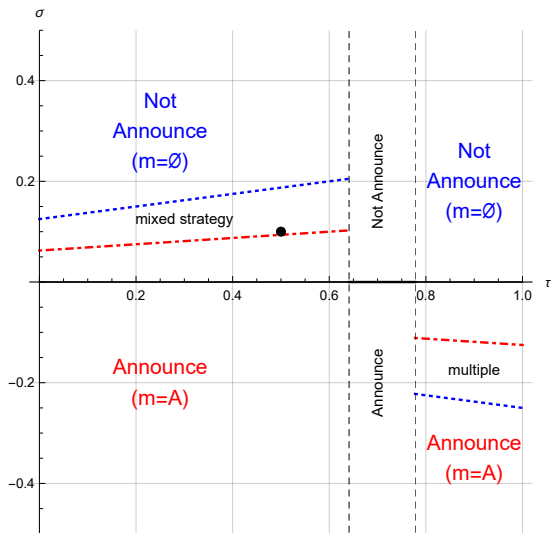
Middle: Gal-Or benchmark (intermediate τ)

- R&D channel inactive; depends only on σ

Right region: Reversed pattern (high τ)

- Even substitutes industries may stay silent to avoid triggering R&D
- **New insight:** Boundary shifts *downward*: some substitutes now conceal

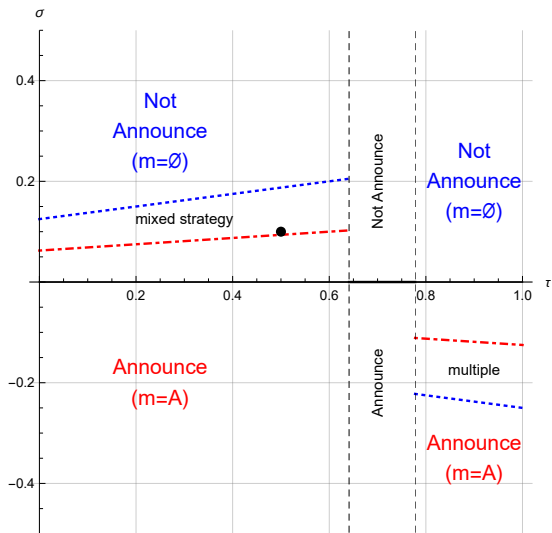
Equilibrium Outcomes and Predictions (3/4)



Key takeaway:

- The R&D deterrence channel *expands* the disclosure region for low τ and *shrinks* it for high τ
- The classic Gal-Or threshold ($\sigma = 0$) is a special case

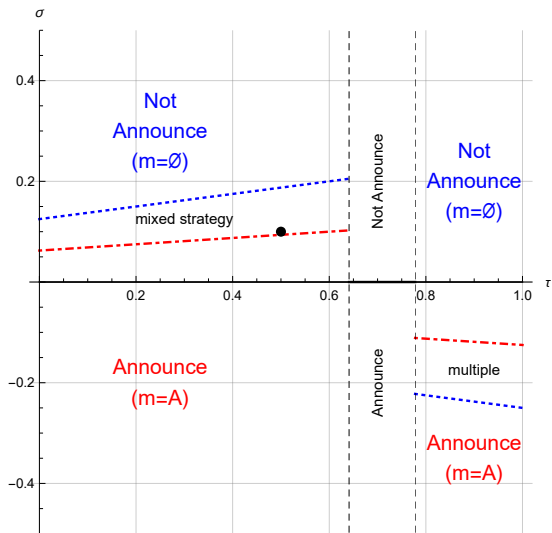
Equilibrium Outcomes and Predictions (4/4)



Testable predictions:

P1. Announcements decrease in σ .

Equilibrium Outcomes and Predictions (4/4)

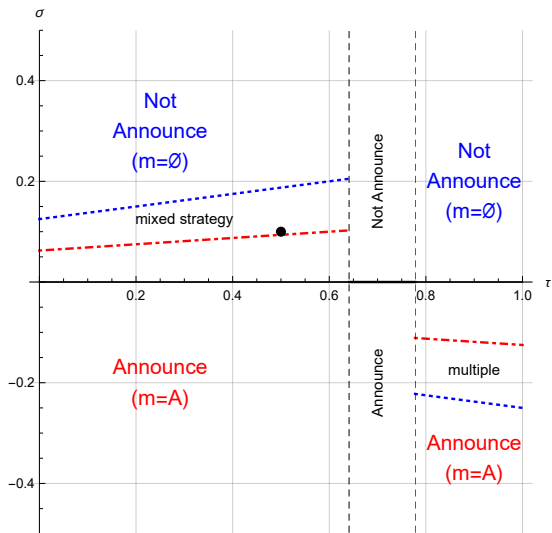


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P2. Announcements decrease in τ .

Equilibrium Outcomes and Predictions (4/4)



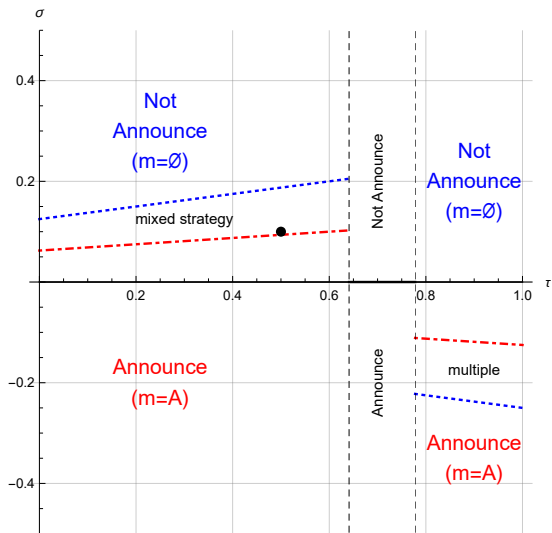
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P3. Negative association of $|\sigma|$ with disclosure for lower τ , positive or no association for higher τ
 $\implies$ positive interaction term.

Equilibrium Outcomes and Predictions (4/4)



Testable predictions:

P1. Announcements decrease in σ .

P2. Announcements decrease in τ .

P3. Negative association of $|\sigma|$ with disclosure for lower τ , positive or no association for higher τ
 $\implies$ positive interaction term.

P4. Negative association of τ with disclosure is strongest for intermediate σ .

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Empirical Strategy: Overview

- Test predictions using different U.S. firm- and industry-level data
- **Three disclosure proxies** (dependent variables):
 1. Mentions of pending/filed patents in SEC 10-K filings (firm level)
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- **Two key independent variables:**
 - Nature of competition $\hat{\sigma}$: proxy following Kedia (2006)
 - R&D correlation τ : within-industry patent concentration (HHI by CPC class)
- Both $\hat{\sigma}$ and τ measured as of 2014 (pre-determined)

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- Both $\hat{\sigma}$ and τ measured as of 2014 (pre-determined)
- **Controls:** share of small firms, patent grant rate γ , number of assignees

Measuring the Nature of Competition ($\hat{\sigma}$)

- We need to estimate $\pi''_{ij} = \frac{\partial^2 \pi^i}{\partial x_i \partial x_j}$, the sign of which determines the nature of competition (Bulow et al., 1985)
- **Challenge:** We don't observe firms' strategic variables (x) — only sales ($s = p \cdot q$)

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- **Challenge:** We don't observe firms' strategic variables (x) — only sales ($s = p \cdot q$)
- **Kedia's (2006) insight:** Use the total differential of marginal profit to recover the sign of π''_{ij} from observable data. Proxy x with s and regress:

$$d \left[\frac{\partial \pi_i}{\partial s_i} \right] = \underbrace{\beta_1 s_i ds_i + \beta_2 ds_i}_{\text{own-output effects}} + \underbrace{\beta_3 s_i ds_{-i} + \beta_4 ds_{-i}}_{\text{competitive interaction}}$$

→ LHS: change in firm i 's marginal profitability (quarterly Δ net income / Δ sales)

→ ds_{-i} : change in total sales of all other firms in the same industry

- The linear combination $\hat{\beta}_3 \bar{s}_i + \hat{\beta}_4$ approximates π''_{ij} and, for linear demand, correctly identifies its *sign*

Measuring $\hat{\sigma}$: From Firms to Industries

- **Estimation:** OLS for each firm using quarterly Compustat data (10-year window, 2005–2014)
- **Firm-level classification** based on the sign of $\hat{\beta}_3 \bar{s}_i + \hat{\beta}_4$:
 - Significantly positive (F -test, 10% level) → $\tilde{\sigma}_i = +1$ (complements)
 - Significantly negative → $\tilde{\sigma}_i = -1$ (substitutes)
 - Not significant → no strategic interaction
- **Industry-level aggregation:** $\hat{\sigma} = \mathbb{E}(\tilde{\sigma}_i) \in [-1, 1]$
 - Average across all firms in a 6-digit NAICS industry
 - Baseline: 2014 value → pre-determined relative to outcomes (2015–2022)
- **Result:** 384 industries with ≥ 2 publicly traded firms:
 - 137 complements ($\hat{\sigma} > 0$), 130 substitutes ($\hat{\sigma} < 0$), 117 undefined

Measuring R&D Correlation (τ)

- Within-industry **Herfindahl–Hirschman Index** over 4- or 6-digit CPC technology classes
- Computed from patents filed by Compustat firms in a given NAICS industry
- Higher concentration $\rightarrow$ firms patent in similar technology areas $\rightarrow$ higher τ
- Baseline: 2014 value (10-year window, 2005–2014) $\rightarrow$ pre-determined
- Available for 512 six-digit NAICS industries
- Full overlap of $\hat{\sigma}$ and τ : 299 industries
- With all variables non-missing: 187 industries

Disclosure Proxy 1: SEC 10-K Filings

- Publicly listed firms are required to file annual 10-K reports with the SEC
- Whether to mention pending patent applications is at the firm's discretion
- **Construction:**
 - Pre-processed filings from the Notre Dame Software Repository for Accounting and Finance
 - Search main text and exhibits for “patent” and either “filed” or “pending”
 - Exclude occurrences only in the exhibits section
 - Code as disclosure dummy = 1 if both terms appear in same paragraph
- **Advantages:**
 - Observe the universe of public firms, *including non-disclosers*
 - Firm-level analysis with $\approx 27,000$ firm-year observations (2015–2022)
- **Limitation:** Motivation for mentioning patents may vary (legal, strategic, investor relations)

Disclosure Proxy 2: Press Releases

- Any firm can issue a press release on a topic of its choosing.
- **Construction:**
 -
 - Searched NexisUni for “patent application” OR “patent applications” AND “files” or “filed”
 - Source types: press releases and newswires, region: United States
 - ≈30,000 results → classified by LLM whether actually announcing a pending application
 - Extracted identity of the issue and applicant, matched with ORBIS to obtain NAICS
 - Final sample: 1,797 announcements in 209 six-digit NAICS industries
- **Advantage:** most direct window on the disclosure decision
- **Limitations:**
 - Unable to identify firms that *could have* announced but chose not to → **industry-level** analysis
 - An approach mainly taken by younger firms

Disclosure Proxy 3: Early Publication Requests

- Every applicant can ask the USPTO to publish a pending application prematurely
- **Construction:**
 - USPTO Patent Examination Research Dataset (Graham et al., 2018)
 - Requests to publish *before* the statutory 18-month deadline
 - Mapped to NAICS via Lybbert and Zolas (2014) concordance; manufacturing industries
 - industry level, 2015–2021
- **Advantage:** easy to construct
- **Limitation:** EPRQs disclose patent *content* (not just existence) — modified model yields same predictions

Estimation details

Identification

Distribution of $\hat{\sigma}$

Summary Statistics (All Variables Non-Missing)

Sample: 187 six-digit NAICS industries with complete data on all variables.

	Mean	SD	Median	Min	Max
<i>6-digit NAICS industry averages (N = 187):</i>					
SEC filings	46.52	221.92	13.00	0.00	2,872.36
Press releases (with zeros added)	1.39	4.06	0.00	0.00	31.00
EPRQs	7.39	13.13	2.02	0.00	97.00
No. of assignees	235.10	462.14	48.53	0.01	3,763.56
<i>Firm-level data: (N = 17,094)</i>					
SEC filing dummy:	0.70	0.46	1	0	1
Competition $\hat{\sigma}$	0.03	0.49	0.00	-1.00	1.00
R&D correlation τ	0.05	0.12	0.01	0.00	1.00
Grant rate γ	0.70	0.12	0.73	0.38	1.00
Share of firms < 10 emp.	0.55	0.21	0.52	0.02	0.95

- $\hat{\sigma}$ and τ pre-determined (2014 values, 10-year estimation windows)

Results: SEC 10-K Filings (Firm Level)

Estimation: OLS. DV: dummy = 1 if firm mentions pending/filed patents in 10-K. Year and state FE included.

- **Prediction 1:** $\hat{\sigma}$ is negative and significant (-0.136^{**})
- **Prediction 2:** τ is negative and significant (-0.685^{**})
- **Prediction 3:** $|\hat{\sigma}| \times \tau$ interaction: 0.957^{**}
- **Prediction 4:** marginal effect of τ strongest at medium $\hat{\sigma}$ (-2.031^{**})
- Sample: $\approx 15,000$ patenting firms (i.e., potential announcers)
- **All four predictions supported with statistical significance**
- Employment (log) negative and significant.

SEC 10-K Filings: Main Effects (P1 & P2)

Estimation: OLS. DV: dummy = 1 if firm mentions pending/filed patents in 10-K.

	(1)	(2)
$\hat{\sigma}$	-0.136** (0.059)	-0.131** (0.061)
τ		-0.685** (0.326)
N Obs.	15,316	15,019
R^2	0.11	0.12

All regressions contain year and state FE and log. employment.

Standard errors in parentheses clustered at industry level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

- **P1** confirmed: $\hat{\sigma}$ negative and highly significant → more disclosure under substitutes
- **P2** confirmed: τ negative and highly significant → more disclosure when R&D

SEC 10-K Filings: Interaction Effects (P3 & P4)

Hypothesis tested:	(3) P3	(4) P4
$\hat{\sigma}$		-0.491* (0.296)
τ	-0.808*** (0.227)	-0.540* (0.287)
$ \hat{\sigma} $	-0.290** (0.227)	
$ \hat{\sigma} \times \tau$	0.957** (0.448)	
Med. $\hat{\sigma} \times \tau$		-2.031** (0.949)
High $\hat{\sigma} \times \tau$		0.331 (0.293)
N Obs.	15,019	15,019
R^2	0.14	0.20

Marginal effects of τ in col. (4): medium $\hat{\sigma}$: -2.571*** | high $\hat{\sigma}$: -0.209**

Consistent Results with Alternative Disclosure Proxies

- We replicate the analysis with two industry-level proxies:
 1. **Press releases** announcing pending patents (log-OLS, 188–267 industries)
 2. **Early Publication Requests** at the USPTO (log-OLS, 187–193 industries)
- **All four predictions are supported**, with point estimates of consistent sign
- Statistical significance is naturally lower due to sample sizes two orders of magnitude smaller than the SEC firm-level analysis
- Interaction effects (P3 & P4) are significant across all three proxies:
 - $|\hat{\sigma}| \times \tau$: +0.957** (SEC) +3.267*** (press) +2.941*** (EPRQs)
 - Marg. effect of τ at med. $\hat{\sigma}$: -2.571*** -1.382** -1.838***

Robustness

- **Alternative specifications for SEC filings:**

- Higher-level (2 or 3 digits) industry fixed effects: results stable though less significant
- Clustered standard errors at 4-digit NAICS: broadly robust
- Equivalent results OLS/Logistic/Fractional regression

- **Alternative level of analysis:**

- SEC filings aggregated to industry level: point estimates consistent, significance lower (smaller sample)

- **Alternative variable construction:**

- Different time windows for $\hat{\sigma}$ (annual values, sample-period averages)
- 4-digit CPC subclasses for τ
- Alternative CSM specifications
- Results remain qualitatively unchanged

Roadmap

Motivation

Model

Equilibrium Results

Comparative Statics and Predictions

Empirical Evidence

Conclusion

Summary

- **Setting:** Patent applications are secret for 18 months. Firms can voluntarily disclose their existence.
- **Model:** The leader trades off
 - Short-run: the value of secrecy in product-market competition
 - Long-run: the value of deterring (or cost of triggering) rival's R&D
- **Key results beyond the classic benchmark:**
 - Low R&D correlation: disclosure even under some strategic complements
 - High R&D correlation: non-disclosure even under some strategic substitutes
- **Empirical evidence:** Three distinct proxies consistently support all four predictions
 - First empirical evidence for the classic theoretical prediction (Gal-Or, 1986)

Discussion and Broader Implications

- **Managerial implications:**

- The decision to publicize a patent filing is not just about IP, but about competitive strategy
- Firms should consider both the nature of their competitive environment (substitutes vs. complements) and how similar their R&D programs are

- **Policy relevance:**

- Patent publication rules (e.g., AIPA 1999) govern voluntary disclosure incentives
- Understanding strategic disclosure informs the design of mandatory disclosure policies

- **Extensions:**

- Endogenous R&D investment by the leader
- “Completing” the game tree by allowing firms without technology to announce
 - is this a viable strategy, and why (not)?

Competition and the Strategic Disclosure of Innovation:
Theory and Evidence from Patent Applications

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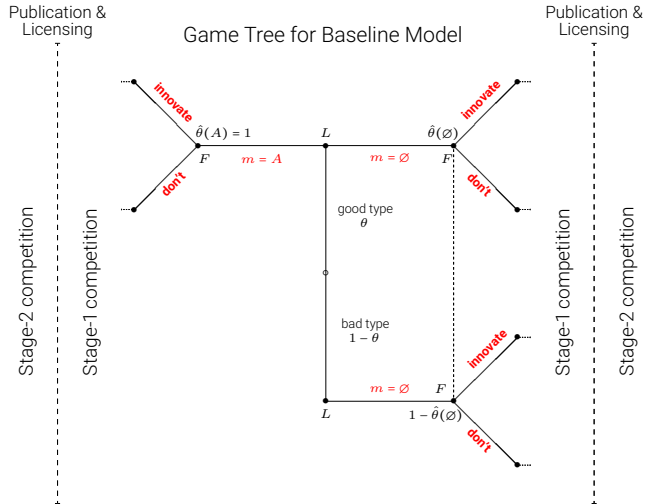
Appendix

Motivating Examples: Quotes

SALEM, NH – (Marketwired - May 14, 2015) – ProPhotonix Limited, a designer and manufacturer of LED illumination systems and laser diode modules [...] today announces that it has filed a patent application on its unique heat sink for optical modular array assemblies. ...

Vancouver, British Columbia – (Newsfile Corp. - September 30, 2015) – PyroGenesis Canada has filed a provisional patent for a one step process using plasma arc for producing high purity silicon from silica. ...

Formal Model: Game Tree



Formal Model: Bayesian Updating

- Prior belief that leader is good type: $\Pr(i = G) = \theta$
- Leader announces with probability μ_G : strategy profile $\bar{\mu} = (\mu_G, 0)$
- Follower's posterior beliefs:

$$\hat{\theta}_1 \equiv \hat{\theta}(m|\bar{\mu}) = \begin{cases} 1 & \text{if } m = A \\ \frac{\theta(1 - \mu)}{1 - \theta\mu} & \text{if } m = \emptyset \end{cases}$$

Formal Model: Technology Spillovers

- Follower invests in R&D at cost K , succeeds with probability $\tilde{\theta}(\hat{\theta}_1)$
- R&D outcomes are correlated: parameter $\tau \in [0, 1]$

$$\tilde{\theta}(\hat{\theta}_1) = (1 - \tau)\theta + \tau\hat{\theta}_1$$

- Under complete information:

$$\tilde{\theta}_B = (1 - \tau)\theta, \quad \tilde{\theta}_G = (1 - \tau)\theta + \tau = \tilde{\theta}_B + \tau$$

Formal Model: Stage-1 Profits

- Leader's stage-1 profits as a function of the follower's beliefs $\hat{\theta}_1$:

$$\tilde{\pi}_i(\hat{\theta}_1) = \begin{cases} \tilde{\pi}_B = (1 - \sigma\hat{\theta}_1)\pi_{BB} \\ \tilde{\pi}_G = (1 - \sigma\hat{\theta}_1 + \sigma)\pi_{GB} \end{cases}$$

- $\sigma > 0$ (complements): profits decrease when follower believes leader is strong
- $\sigma < 0$ (substitutes): profits increase when follower believes leader is strong

Formal Model: Intellectual Property

- Patents are probabilistic: granted/upheld with probability $\gamma \in [0, 1]$
- Follower's technology infringes with probability $\eta \in [0, 1]$; let $\beta \equiv (1 - \eta)\gamma$
- License fee (Nash bargaining): $\lambda = \frac{\pi_{GB}}{2}$
- Two motives for follower's R&D:
 - Competitive advantage if leader is bad: $\psi_{F|B} = \pi_{GB} - \pi_{BB}$
 - License-fee savings if leader is good: $\psi_{F|G} = \frac{\beta\pi_{GB}}{2}$

Formal Model: Follower's R&D Decision

- Net benefits from R&D (Lemma 2):

$$R(\hat{\theta}_1) = \hat{\theta}_1 \tilde{\theta}_G \psi_{F|G} + (1 - \hat{\theta}_1) \tilde{\theta}_B \psi_{F|B} - K$$

- Follower invests if $R(\hat{\theta}_1) \geq 0$
- Net benefits are *decreasing* in $\hat{\theta}_1$ (announcement deters R&D) iff:

$$(1 - \tau)\theta[\psi_{F|G} - \psi_{F|B}] + \tau\psi_{F|G} < 0$$

- Necessary condition: $\psi_{F|G} < \psi_{F|B}$ (competition effect > license savings)

Result 1: When Announcement Deters R&D (Low τ)

[back](#)

When the follower's R&D incentives *decrease* after an announcement:

- **Separating equilibrium:** Leader announces ($m = A$)
 - Arises when the short-run cost of disclosure is small relative to the long-run benefit of deterring R&D
 - Holds broadly under strategic substitutes, and extends into strategic complements for low enough τ
- **Pooling equilibrium:** Leader stays silent ($m = \emptyset$)
 - Arises when the short-run cost of disclosure (under complements) outweighs the R&D deterrence benefit
- **Mixed strategy:** Leader randomizes for intermediate cases

Result 1: When Announcement Deters R&D (Low τ)

[back](#)

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 - Arises when the short-run cost of disclosure (under complements) outweighs the R&D deterrence benefit
- **Mixed strategy:** Leader randomizes for intermediate cases

New insight vs. Gal-Or (1986): The R&D deterrence motive can push disclosure *beyond* strategic substitutes into the complements region.

[Formal conditions](#)[Proof sketch](#)

Formal Model: Equilibrium Conditions (Scenario 1)

Proposition 1 (Pure strategies, announcement deters R&D). Let $R(\theta) \geq 0 > R(1)$:

1. Separating equilibrium ($m = A$) if $\frac{2\sigma}{(1-\tau)\theta + \tau} \leq \beta$
2. Pooling equilibrium ($m = \emptyset$) if $\frac{2\sigma(1-\theta)}{(1-\tau)\theta + \tau} \geq \beta$

Proposition 2 (Mixed strategies):

$$\mu_G^* = \frac{1}{\theta} - \frac{1-\theta}{\theta} \cdot \frac{2\sigma}{[(1-\tau)\theta + \tau]\beta}$$

Result 2: When Announcement Has No Effect on R&D (Intermediate τ)

When the follower always invests or never invests regardless of announcement:

- The long-run channel is inactive $\rightarrow$ only the short-run effect matters
- **Strategic substitutes** ($\sigma < 0$): Leader announces
- **Strategic complements** ($\sigma > 0$): Leader stays silent

Result 2: When Announcement Has No Effect on R&D (Intermediate τ)

When the follower always invests or never invests regardless of announcement:

- The long-run channel is inactive $\rightarrow$ only the short-run effect matters
- **Strategic substitutes** ($\sigma < 0$): Leader announces
- **Strategic complements** ($\sigma > 0$): Leader stays silent
- This is exactly the classic Gal-Or (1986) result:
 - $\rightarrow$ Cournot-type competition $\rightarrow$ disclosure
 - $\rightarrow$ Bertrand-type competition $\rightarrow$ secrecy

Formal conditions (P3 & P4)

Result 3: When Announcement Triggers R&D (High τ)

When the follower's R&D incentives *increase* after an announcement:

- Strong technology spillovers: the follower learns “if they succeeded, I probably can too”
- **Separating equilibrium:** Leader announces ($m = A$) only if strategic substitutes are strong enough to justify triggering R&D
- **Pooling equilibrium:** Leader stays silent ($m = \emptyset$) when the cost of triggering R&D outweighs the short-run benefit

Result 3: When Announcement Triggers R&D (High τ)

When the follower's R&D incentives *increase* after an announcement:

- Strong technology spillovers: the follower learns “if they succeeded, I probably can too”
- **Separating equilibrium:** Leader announces ($m = A$) only if strategic substitutes are strong enough to justify triggering R&D
- **Pooling equilibrium:** Leader stays silent ($m = \emptyset$) when the cost of triggering R&D outweighs the short-run benefit

New insight vs. Gal-Or (1986): Even under strategic substitutes (where the classic model predicts disclosure), the leader may choose secrecy to avoid encouraging the follower's R&D.

Formal conditions (P3 & P4)

Formal Model: Equilibrium Conditions (Scenario 2)

Proposition 3 (R&D unaffected by announcement):

- $m = A$ if $\sigma \leq 0$; $m = \emptyset$ if $\sigma \geq 0$ (Gal-Or benchmark)

Proposition 4 (Pure strategies, announcement triggers R&D). Let $R(1) \geq 0 > R(\theta)$:

1. Separating equilibrium ($m = A$) if $-\frac{2\sigma}{(1-\tau)\theta + \tau} \geq \beta$
2. Pooling equilibrium ($m = \emptyset$) if $-\frac{2\sigma(1-\theta)}{(1-\tau)\theta + \tau} \leq \beta$

[Back to Result 2](#)

[Back to Result 3](#)

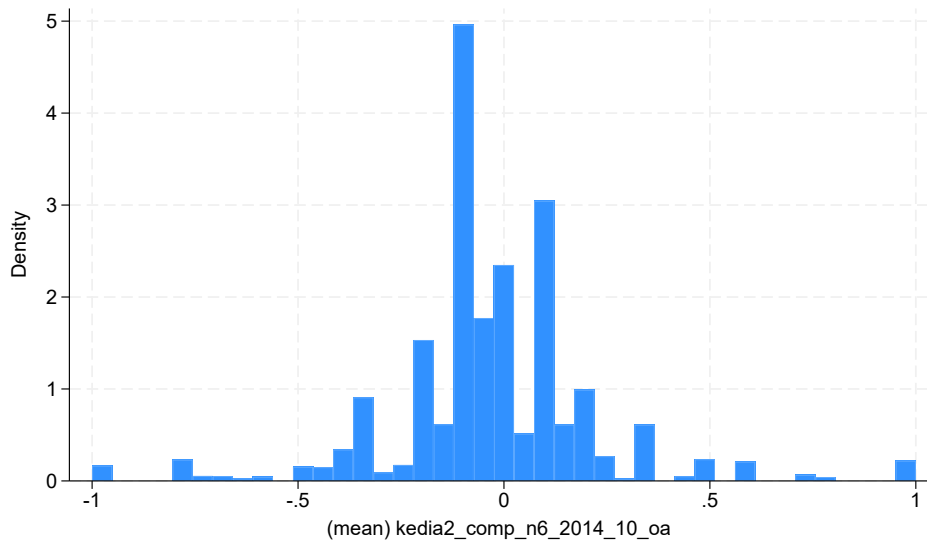
Follower's Complete Payoff Characterization

- Follower's expected net payoffs given $m \in \{A, \emptyset\}$ and $r \in \{0, 1\}$:

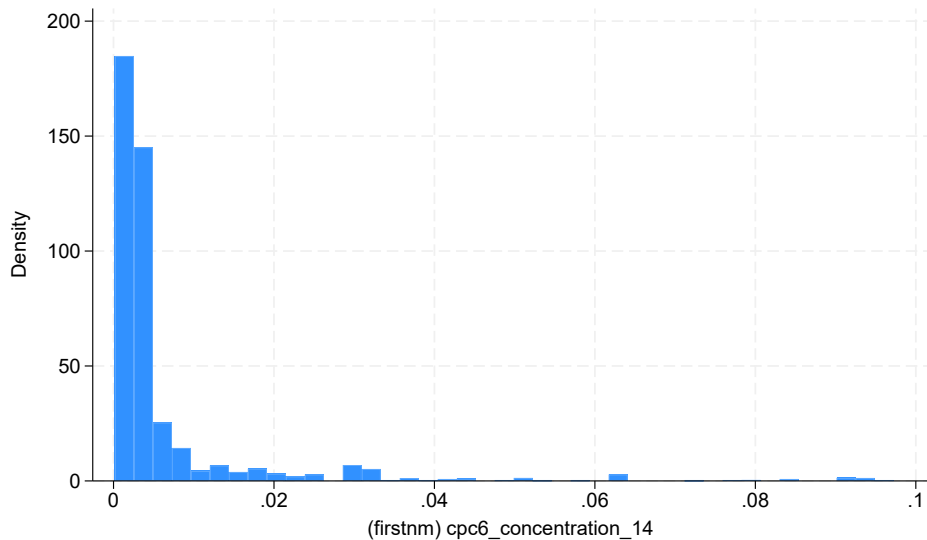
$$\begin{aligned}\pi_F(m, r) = & \hat{\theta}(m|\bar{\mu}) \left[\pi_{GG} - \gamma\lambda + r\tilde{\theta}_G\psi_{F|G} \right] \\ & + (1 - \hat{\theta}(m|\bar{\mu})) \left[\pi_{BB} + r\tilde{\theta}_B\psi_{F|B} \right] - rK\end{aligned}$$

- $\psi_{F|G} = \frac{\beta\pi_{GB}}{2}$: license-fee savings from successful R&D (if leader is good)
- $\psi_{F|B} = \pi_{GB} - \pi_{BB}$: competitive advantage from R&D (if leader is bad)

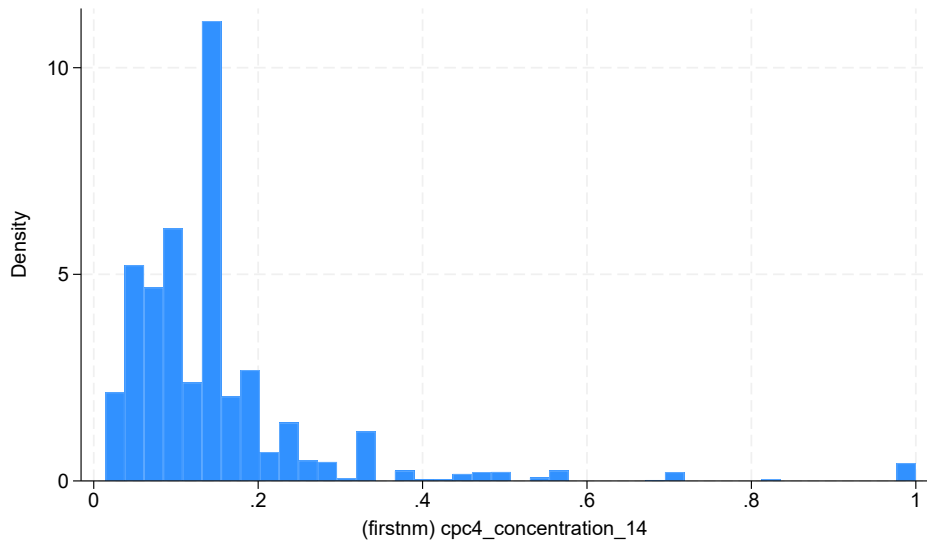
Distribution of $\hat{\sigma}$ Across Industries



Distribution of τ Across Industries (6-Digit CPC)



Distribution of τ Across Industries (4-Digit CPC)



Summary Statistics

	Mean	SD	Median	N
<i>Dependent variables (counts):</i>				
Press releases	3.76	9.00	1.00	209
SEC filings	16.86	113.81	3.00	840
EPRQs	6.97	13.52	1.82	349
<i>Independent variables:</i>				
$\hat{\sigma}$	0.01	0.51	0.00	384
τ	0.07	0.15	0.02	512
γ	0.68	0.13	0.71	827
No. assignees	86.21	264.48	4.70	888

Press Releases: Main Effects (P1 & P2)

Estimation: log-OLS. DV: count of press releases per 6-digit NAICS.

	(1)	(2)
$\hat{\sigma}$	-0.151 (0.110)	-0.170 (0.111)
τ	-0.556* (0.320)	-0.222 (0.326)
N Obs.	194	194
R^2	0.02	0.05

- **P1:** $\hat{\sigma}$ negative but not significant at conventional levels (small sample: 194 industries)
- **P2:** τ negative and significant at 10% without controls

Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Press Releases: Interaction Effects (P3 & P4)

	(3)	(4)
Hypothesis tested:	P3	P4
$\hat{\sigma}$		-0.515** (0.235)
τ	-1.776** (0.731)	-0.224 (0.371)
$ \hat{\sigma} $	-0.290** (0.122)	
$ \hat{\sigma} \times \tau$	3.267*** (1.227)	
Share of firms < 10 emp.	0.548** (0.214)	0.836*** (0.296)
N Obs.	269	188
R^2	0.05	0.13

Marginal effects of τ in col. (4): medium $\hat{\sigma}$: -1.382** | high $\hat{\sigma}$: 2.454**

Log-OLS. Robust SE in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Early Publication Requests: Main Effects (P1 & P2)

Estimation: log-OLS. DV: count of EPRQs per 6-digit NAICS industry.

	(1)	(2)
$\hat{\sigma}$	-0.292* (0.164)	-0.215 (0.136)
τ	-1.621*** (0.460)	-1.327*** (0.501)
Share of firms < 10 emp.	2.225*** (0.439)	
N Obs.	187	193
R^2	0.17	0.22

- **P1:** $\hat{\sigma}$ negative and significant at 10%
- **P2:** τ negative and highly significant (-1.621^{***} , -1.327^{***})

Early Publication Requests: Interaction Effects (P3 & P4)

	(3)	(4)
Hypothesis tested:	P3	P4
$\hat{\sigma}$		-0.397 (0.286)
τ	-2.534*** (0.596)	-0.403 (0.452)
$ \hat{\sigma} $	-0.459* (0.249)	
$ \hat{\sigma} \times \tau$	2.941*** (1.057)	
Share of firms < 10 emp.	2.375*** (0.463)	2.099*** (0.412)
N Obs.	187	187
R^2	0.17	0.45

Marginal effects of τ in col. (4): medium $\hat{\sigma}$: -1.838*** | high $\hat{\sigma}$: 2.977***

Log-OLS. Robust SE in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Measuring $\hat{\sigma}$: Theoretical Foundation

- Consider a duopoly: firm i 's profit $\pi^i = \pi(x_i, x_j)$, where x_i, x_j are the firms' actions
- The cross-partial derivative $\pi''_{ij} = \frac{\partial^2 \pi^i}{\partial x_i \partial x_j}$ determines the nature of competition (Bulow et al., 1985):
 - $\pi''_{ij} < 0$: strategic substitutes (rival increases $x_j \rightarrow i$'s marginal profit falls)
 - $\pi''_{ij} > 0$: strategic complements (rival increases $x_j \rightarrow i$'s marginal profit rises)
- **Problem:** We cannot directly observe π''_{ij} because
 1. We do not observe the strategic variable x (quantity or price)
 2. We only observe revenues (= price $\times$ quantity), i.e., sales s
- **Solution** (Kedia, 2006): proxy x with sales s and use the total differential of marginal profit to estimate the sign of π''_{ij}

Measuring $\hat{\sigma}$: Deriving the Estimating Equation

- Start with the first derivative of profit:

$$\frac{\partial \pi^i}{\partial x_i} = D'_i(x_i, x_j) x_i + D(x_i, x_j) - C'_i(x_i)$$

where D is the demand function and C is the cost function

- Take the total differential and group by changes in own and rival actions:

$$d \left[\frac{\partial \pi_i}{\partial s_i} \right] = \beta_1 s_i ds_i + \beta_2 ds_i + \beta_3 s_i ds_{-i} + \beta_4 ds_{-i}$$

- The β 's depend on the demand and cost functions:

→ $\pi''_{ij} = \beta_3 x_i + \beta_4$ captures the strategic interaction

→ Proxying x_i with sales s_i : $\hat{\beta}_3 \bar{s}_i + \hat{\beta}_4$ approximates π''_{ij}

- Key result:** For linear demand, this correctly identifies the *sign* of π''_{ij} (but not magnitude)

Measuring $\hat{\sigma}$: Estimation and Aggregation

- **Data:** Quarterly sales and net income from Compustat
 - Marginal profit approximated as: $\frac{\partial \pi_i}{\partial s_i} \approx \frac{\Delta \pi_i}{\Delta s_i}$ (quarterly changes in net income / sales)
 - ds_{-i} : change in total sales of all other firms in the same industry
- **Estimation:** OLS regression for each firm using 10 years of quarterly data (40 observations per firm)
- **Classification at firm level:**
 - If $\hat{\beta}_3 \bar{s}_i + \hat{\beta}_4 > 0$ and significant (F -test, 10% level) → $\tilde{\sigma}_i = +1$ (complements)
 - If $\hat{\beta}_3 \bar{s}_i + \hat{\beta}_4 < 0$ and significant → $\tilde{\sigma}_i = -1$ (substitutes)
 - If not significant → no strategic interaction
- **Aggregation to industry level:**
 - $\hat{\sigma} = \mathbb{E}(\tilde{\sigma}_i) \in [-1, 1]$: average across all firms in a 6-digit NAICS industry
 - Baseline: 2014 value (10-year window, 2005–2014) → pre-determined

Measuring $\hat{\sigma}$: Identification and Limitations

- **Identification challenge:**

- Reaction functions shift with cost and demand shocks
- We observe equilibrium points, not the reaction functions themselves

- **Kedia's solution:** Estimate each firm's reaction against the *aggregate* reaction function of all other firms in the industry

- Individual rivals' reaction functions shift with idiosyncratic errors
- The aggregate reaction function is more stable → aids identification

- **Limitations:**

- Identifies the sign but not the magnitude of strategic interaction
- Requires at least two publicly traded firms per industry
- Assumes linear demand functions for sign consistency

- **Robustness:** Results stable across annual values, sample-period averages, and shorter estimation windows

Proof Sketch: Proposition 1 (Separating Equilibrium)

- Separating: $\mu_G = 1$. Leader announces, follower does not invest.
- Leader's payoffs from announcing: $\tilde{\Pi}_G(\hat{\theta}(A|(1,0))) = \pi_{GB} + \pi_{GG} + \gamma\lambda$
- If leader deviates (does not announce):
 - $\hat{\theta}(\emptyset|(1,0)) = 0$, follower invests
 - Leader's payoffs: $(1 + \sigma(1 - \theta))\pi_{GB} + \pi_{GG} + \gamma\lambda - \rho_G\psi_{F|G}$
- Separating equilibrium exists if $\tilde{\Pi}_G(A) \geq \tilde{\Pi}_G(\emptyset)$
- Rearranging yields condition: $\frac{2\sigma}{(1-\tau)\theta+\tau} \leq \beta$